GAUGE INVARIANT FIELD STRENGTH CORRELATORS IN QCD.

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Gauge invariant correlators in QCD are studied on the lattice. A systematic determination of the correlation lengths for gluon field strength correlators and quark correlators is made. The measurement of the gluon and quark condensates is discussed.

1 Introduction

The gauge invariant gluon field strength correlators are defined as

$$\mathcal{D}_{\mu\nu\rho\sigma}(x,C) = \langle 0|T\left(G^a_{\mu\nu}(x)S^{ab}_{adj,C}(x,0)G^b_{\rho\sigma}(0)\right)|0\rangle \quad (1)$$

with $S^{ab}_{adj,C}(x,0)$ the parallel transport from 0 to x along the path C, in the adjoint representation

$$S_{adj,C}(x,0) = P\exp\left(i\int_{0,C}^{x} A^a_\mu(y)T^a_{adj}\,dy^\mu\right) \quad (2)$$

T^a_{adj} are the group generators in the adjoint representation.

$\mathcal{D}_{\mu\nu\rho\sigma}(x,C)$ depends on the choice of C: in what follows we will take for C a straight line, and drop the dependence on C.

Higher correlators are usually defined by parallel transport to a fixed point x_0.

Fermion correlators are defined as

$$S_i(x) = \langle 0|T\left(\bar\psi(x)S_{fund}(x,0)M^i\psi(0)\right)|0\rangle \quad (3)$$

M^i is a generic element of the Clifford algebra of the γ matrices. S_{fund} is the analog of the transport (2) in the fundamental representation, and again we have once and for all assumed for the path C a straight line.

By use of general covariance arguments[1,2] $\mathcal{D}_{\mu\nu\rho\sigma}$ can be parametrized in terms of two independent invariant form factors $\mathcal{D}(x^2)$ and $\mathcal{D}_1(x^2)$

$$\begin{aligned}\mathcal{D}_{\mu\nu\rho\sigma}(x) &= (g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\left[\mathcal{D}(x^2)+\mathcal{D}_1(x^2)\right] + \quad (4)\\ &+(x_\mu x_\rho g_{\nu\sigma} - x_\mu x_\sigma g_{\nu\rho} - x_\rho x_\nu g_{\mu\sigma} + x_\nu x_\sigma g_{\mu\rho})\frac{\partial \mathcal{D}_1}{\partial x^2}\end{aligned}$$

Similarly one can prove that all the correlators (3) vanish by T, P invariance, except the correlator S_0 corresponding to $M^i = I$, the identity matrix

$$S_0(x) = \langle 0|T\left(\bar\psi(x)S_{fund}(x,0)\psi(0)\right)|0\rangle \quad (5)$$

and the vector correlator ($M^i = \gamma^\mu$), with γ^μ in the direction of x

$$\frac{x^\mu}{|x|}S_V(x) = \langle 0|T\left(\bar\psi(x)S_{fund}(x,0)\gamma^\mu\psi(0)\right)|0\rangle \quad (6)$$

The physical interest of the above correlators stems from the basic idea of the ITEP sum rules[3]: the long distance modes of QCD are described by a slowly varying background, made e.g. of instantons, on which high momentum perturbative fluctuations are superimposed. In the O.P.E. low modes generate the condensates, and high frequency modes the corresponding coefficient functions $C_n(x)$

$$\begin{aligned}T\left(j_\mu(x)j_\nu(0)\right) &\simeq \sum_n C_n(x)O_n \quad (7)\\ &= C_I(x)I + C_G(x)G_{\mu\nu}(0)G_{\mu\nu}(0) + \\ &+ C_\psi(x)\sum m_f\bar\psi_f(0)\psi_f(0) + \ldots\end{aligned}$$

As is well known expressing the left hand side in terms of a dispersive integral, relates masses and widths of resonances in $e^+e^- \to hadrons$ to the condensates

$$G_2 = \frac{\beta(g)}{g}\langle 0|G^a_{\mu\nu}(0)G^a_{\mu\nu}(0)|0\rangle \quad , \quad \langle 0|m_f\bar\psi_f(0)\psi_f(0)|0\rangle$$

(SVZ sum rules).

By use of this idea it was proposed in ref.[4,5] that the gluon condensate G_2 could be determined from the spectrum of bound states of heavy $Q\bar{Q}$ systems. If the correlation length of the slow varying field, λ, is much bigger than the typical time of the bound system, then its effect is in all respects a static Stark effect on the levels, and the gluon condensate can be extracted from it.

A more detailed analysis[6] involves $\mathcal{D}_{\mu\nu\rho\sigma}(x)$, and gives a shift depending on the parameter

$$\rho = \lambda \frac{m_q \alpha_s^2}{4} \tag{8}$$

where $4/m_q\alpha_s^2$ is the typical time of the low lying levels of the system, and λ is the correlation length defined as

$$\mathcal{D}(x) \underset{|x|\to\infty}{\simeq} G_2 \exp(-x/\lambda) \tag{9}$$

Measuring λ was the motivation to investigate for the first time $\mathcal{D}_{\mu\nu\rho\sigma}(x)$ on the lattice[7]. The computation was done in quenched $SU(2)$ and gave a surprisingly small value of λ

$$\lambda \simeq 0.16\,\mathrm{fm} \tag{10}$$

shaking the very bases of the SVZ approach.

A stochastic model of the vacuum was subsequently developed[1,2], in which observables are expressed in terms of invariant field strength correlators, and a cluster expansion is made. The basic assumption of the model is that higher order clusters are negligible.

The quark correlator S_0 instead, known also as "non local fermion condensate" enters in the construction of the wave functions of hadrons, in particular in the computation of the pion form factor[8].

A technical breakthrough, the use of cooling or smearing procedures to polish short distance fluctuations, leaving long distance physics unchanged[9], allowed a better determination of correlators[10,11,12].

The physical motivations to study correlators on the lattice are in conclusion

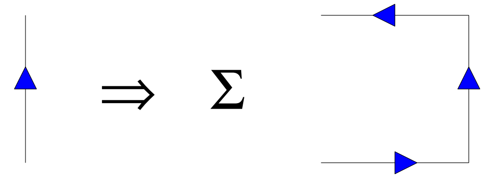
(i) understanding λ, and the basis of SVZ sum rules.

(ii) measuring condensates from first principles.

(iii) more generally providing inputs from first principles to the community of stochastic vacuum practitioners.

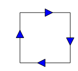
2 Cooling-smearing correlators

Short range fluctuations in lattice configurations can be smoothed off by a local cooling procedure which consists in replacing a link by the sum of the inverse "staples" attached to it



(11)

Since the action density is

$$S \sim \sum_{\mu\nu} \left(1 - \frac{1}{N_c} Tr\Pi_{\mu\nu}\right) \quad \text{with } \Pi_{\mu\nu} =$$ 

(12)

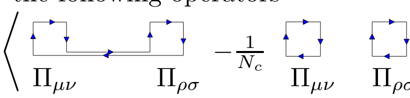
this procedure makes locally $S = 0$. In the euclidean region S plays the role of energy and the replacement (11) locally minimizes S, whence the terminology "cooling".

Like any local procedure cooling n_t times affects distances d by a diffusion process, with

$$d^2 \sim n_t \tag{13}$$

According to eq.(13), n_t can be made sufficiently large to eliminate short range fluctuations but not enough to modify long range correlations[9]. Fluctuations are thus reduced by orders of magnitude without changing long distance physics.

Gauge invariant correlators can be represented on lattice by the following operators[7]

$\mathcal{D}^L_{\mu\nu\rho\sigma} = \Big\langle$  $\Big\rangle$

A series expansion in a gives

$$\mathcal{D}^L_{\mu\nu\rho\sigma}(d) \simeq Z^2 a^4 \mathcal{D}_{\mu\nu\rho\sigma}(d) + \mathcal{O}(a^6) \tag{14}$$

By the cooling procedure $\mathcal{O}(a^6)$ terms disappear and possible renormalizations Z of the field $G_{\mu\nu}$ tend to 1. So finally

$$\mathcal{D}^L_{\mu\nu\rho\sigma}(d) \simeq a^4 \mathcal{D}_{\mu\nu\rho\sigma}(da) \tag{15}$$

In order to use cooling profitably the distance d has to be $\sim 3-4$ lattice spacings at least. At a given β this corresponds to some physical length l_{min}. If we want l_{min} to be small, say 0.1 fm, since the lattice must be at least 1 fm across, the lattice size must be $(10 l_{min})^4$. Going to small distances requires big lattices.

3 Correlators, OPE and renormalons

The SVZ sum rules are based on the OPE of the correlator

$$\begin{aligned}\Pi_{\mu\nu}(q) &= \int d^4x e^{iqx} \langle 0|T\left(j_\mu(x) j_\nu(0)\right)|0\rangle \\ &= \Pi(q^2)\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right)\end{aligned} \tag{16}$$

The OPE gives

$$\Pi(q^2) \simeq c_1 I + c_2 \frac{G_2}{q^4} + \dots \tag{17}$$

The first term corresponds to the perturbative expansion. That expansion, however, is not Borel summable, and is ambiguous by terms of order μ^4/q^4, with μ the renormalization scale (Renormalons). As a consequence the second term in eq.(17) is intrinsically undefined. This is a basic and unavoidable drawback of perturbation theory, reflecting the fact that perturbative vacuum is not the ground state[13].

However, keeping the first few terms in the perturbative expansion of c_1 and c_2 gives a consistent phenomenology[3].

The same happens for the correlators. The OPE of the invariant form factors has the form

$$\mathcal{D}(x^2) \underset{x^2 \to 0}{\simeq} \frac{c_1}{x^4} + c_2 G_2 + \mathcal{O}(x^2) \tag{18}$$

The second term is again undetermined by renormalons coming from the perturbative expansion of c_1. As in the SVZ sum rules we shall assume that the first few terms of the perturbative expansion work, and use it to determine G_2. We shall parametrize the lattice determination of $\mathcal{D}(x^2)$ and $\mathcal{D}^{(1)}(x^2)$ as

$$\frac{1}{a^4}\mathcal{D}_L(x^2) = \frac{a}{|x|^4} e^{-|x|/\lambda_a} + A_0 e^{-|x|/\lambda} \tag{19}$$

$$\frac{1}{a^4}\mathcal{D}_L^{(1)}(x^2) = \frac{a_1}{|x|^4} e^{-|x|/\lambda_a} + A_1 e^{-|x|/\lambda} \tag{20}$$

Eq.(19), (20) obey the OPE eq.(18), and reflect the existence of a mass gap in the theory.

In ref.[7] the first term of eq.(18), (19), (20) was computed in perturbation theory and subtracted. The residual term was an exponential and λ could be extracted from it, giving $\lambda = 0.16$ fm.

In ref.[10,11] quenched $SU(3)$ was studied. The above parametrization gave a good fit to the data (fig.1) with $\lambda = 0.22$ fm and $G_2 = (0.14 \pm 0.02)$ GeV^4, a value larger by an order of magnitude than the phenomenological value[3]. In ref[12] full QCD with 4 staggered fermions was studied, at quark masses $am_q = 0.01$, $am_q = 0.02$. The results are in this case[12]

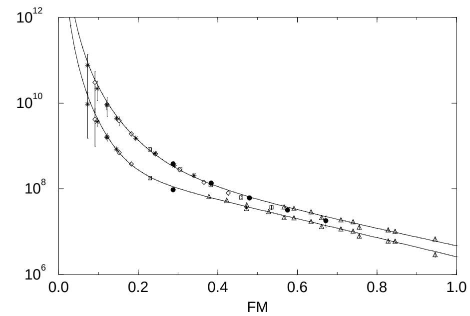


Fig.1 $D_{||}^L/a^4 = \mathcal{D}+\mathcal{D}_1+x^2\partial\mathcal{D}_1/\partial x^2$ and $D_\perp/a^4 = \mathcal{D}+\mathcal{D}_1$ versus x. The lines correspond to the best fit to eq.(19),(20).

$$\begin{array}{lll} am_q = .01 & \lambda = (.34 \pm .02)\,\mathrm{fm} & G_2 = .015 \pm .003\,\mathrm{GeV}^4 \\ am_q = .02 & \lambda = (.29 \pm .02)\,\mathrm{fm} & G_2 = .031 \pm .005\,\mathrm{GeV}^4 \end{array}$$

A common feature to all the determinations is that $|A_1| \simeq A_0/10$. In full QCD the correlation length is bigger, and could agree with the basic philosophy of ref.[3]. Also the value of the condensate G_2 is smaller than the quenched value and agrees with phenomenology.

By use of the relation[14]

$$\frac{d}{dm_f} G_2 = -\frac{24}{b_0}\langle\bar\psi\psi\rangle \qquad b_0 = 11 - \frac{2}{3}N_f$$

one can extrapolate in m_f to the physical value of G_2 getting

$$G_2 \simeq 0.022 \pm .006\ \mathrm{GeV}^4 \tag{21}$$

in agreement with sum rules determination[15].

Similar arguments allow to extract G_2 from the measurement of the average value of the density of action (plaquette): again the level of rigour is the same as for SVZ sum rules[3], at least if only a few terms are kept of the perturbative expansion of the coefficients in the OPE[16,17,18].

A detailed analysis of the behaviour of the correlators at finite temperature, in the vicinity of the deconfining transition T_c was made in ref.[11]. There $O(4)$ invariance is lost reducing to $O(3)$ and 5 independent form factors exist.

The main result is that magnetic correlators are unchanged across T_c, while electric correlators have a sharp drop (fig.2,fig.3).

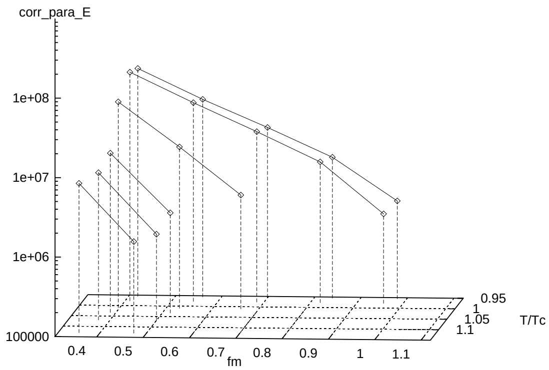


Fig.2 The electric longitudinal correlator versus distance, for different values of T/T_c.

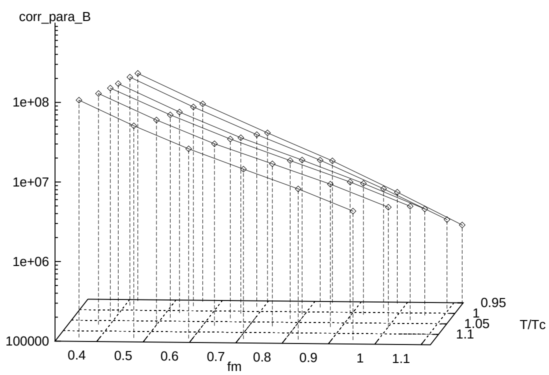


Fig.3 The magnetic longitudinal correlator versus distance, for different values of T/T_c.

The scalar quark correlator

$$S_0(x) = -\langle 0|T(\bar{\psi}(x)S\psi(0))|0\rangle \tag{22}$$

has recently been determined in full QCD and in quenched QCD[19].

A comparison has been made between the determinations in full QCD, at given values of the quark masses $am_q = 0.01$, $am_q = 0.02$ and of the lattice spacing a, and in quenched QCD at the same values of these physical parameters. No difference has been found, within errors, indicating that quark loops do not affect appreciably the quark correlator.

A sensible parametrization for the lattice regulator S_0^L is

$$S_0^L(x) = a^3 A_0 \exp(-x/\lambda_f) + \frac{B_0 a^3}{x^2} \tag{23}$$

Simulations have been performed[19] at $am_q = 0.01$ in full QCD at $\beta = 5.35$ and quenched QCD at $\beta = 6.0$, which correspond both to a lattice spacing $a \simeq 0.10$ fm. No appreciable difference is found and in both cases (fig.4)

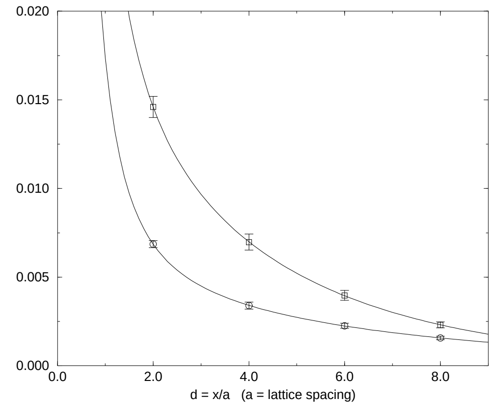


Fig.4 $S_0^L(x)$ versus x. The curve is the best fit to eq.(23).

$$\frac{a}{\lambda_f} = 0.16 \pm 0.04 \qquad am_\pi = 0.26 \pm 0.01 \tag{24}$$

A similar determination at $am_q = 0.02$ $\beta = 5.35$ full QCD, $\beta = 5.91$ quenched, where $a \simeq 0.12$ fm, give indistinguishable results:

$$\frac{a}{\lambda_f} = 0.26 \pm 0.04 \qquad am_\pi = 0.37 \pm 0.01 \tag{25}$$

Putting the two determinations together gives

$$\lambda_f m_\pi = 1.5 \pm 0.3 \tag{26}$$

The typical correlation length is now $\sim 1/m_\pi$ in agreement with the approach of ref.[3].

A determination of A_0 and a study of its relation to the quark condensate $\langle\bar{\psi}\psi\rangle$ is on the way.

4 Discussion

The typical correlation length is rather small for gluon correlators: $\lambda = 0.16$ fm for quenched $SU(2)$, 0.22 fm for quenched $SU(3)$, 0.32 fm for full QCD with 4 flavours. It is bigger for fermion correlators where

$$\lambda_f \simeq (1.5 \pm 0.3) m_\pi^{-1}$$

Condensates can be determined, despite the presence of renormalons, by the same philosophy used for SVZ sum rules. $G_2 = (0.022 \pm 0.006)$ GeV^4 in full QCD is consistent with the determination by sum rules. It is an order of magnitude bigger in quenched QCD, $G_2 = (0.14 \pm 0.02)$ GeV^4.

The behaviour of correlators at the deconfining transition is consistent with expectations.

Our determinations are useful to phenomenology and to test models of QCD vacuum.

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